Charge to Radius Dependency for Conductive Particles Charged by Induction

N. Toljic, G.S.P. Castle, *Life Fellow, IEEE*, and K. Adamiak, *Fellow, IEEE*

***Abstract*— Particle charge is a critical parameter that needs to be determined in order to accurately predict behaviour of a charged particle exposed to electrical forces. The effectiveness of various electrostatic applications depends directly on this charge or, more specifically, the charge to mass ratio. Previous studies report conflicting data for the size dependency of charge. In this paper, the relation between the value of charge on a conductive particle and the particle radius in the process of induction charging is investigated. The results of numerical simulations of a liquid atomization process are presented and a novel approach to the analytical solution of the problem is introduced. It is found that the exponent in the particle charge to radius dependency is equal to two when the particle is in the direct contact with the bulk material. The radius exponent decreases rapidly as the atomizing ligament length is increased. For ligament lengths many times greater than the particle radius, the radius exponent approaches one. Agreement between numerical and analytical results is found to be very good. The results of this study clarify some of the conflicting data in the previously published literature and suggest that the particle charge is practically linearly dependent on radius for atomized liquid particles and proportional to particle surface area for solid particles. In addition it is shown that the charge to mass ratio for liquid particles can be maximized by ensuring the ligament length during atomization is maximum.**

***Index Terms*— charge, charge to mass ratio, induction charge**

I. Introduction

THE value of charge is a critical parameter that needs to be determined in order to accurately predict behavior of a charged particle exposed to electrical forces. The effectiveness of various electrostatic applications depends directly on this parameter. Much theoretical and experimental research has been devoted to assess the value of charge of particles and this is still a subject of ongoing study [1-2]. All the measurement techniques can be classified in two major groups: static and dynamic. In static methods charge is measured directly, whereas the dynamic methods calculate charge indirectly based on the measurement of the particle mobility. Static methods tend to be crude and simple, but they can be easily implemented. Dynamic methods can be more reliable, but they introduce significant complexity to the measurement system. It has been shown in [2] that much conflicting data exists in previous literature regarding the dependence of charge to mass ratio on particle size and it has been suggested that this area needs further experimental and theoretical research.

For the charging of conductive particles, analytical solutions for certain geometrical cases are available. These solutions can typically be presented in the form:

$$q = Ar^n \tag{1}$$

where q is the particle charge, r is the particle radius and A and n are unknown coefficients dependent on the various parameters. For the sphere with radius r in the direct contact with an infinite plane exposed to a uniform electric field, the value of acquired charge is [3]:

$$q = 6.56\pi\varepsilon_0 E_0 r^2 \tag{2}$$

where ε_0 is the electrical permittivity and E_0 the value of the external electric field. Equating the right sides of (1) and (2) yields:

$$A = 6.56\pi\varepsilon_0 E_0 \tag{3}$$

$$n = 2 \tag{4}$$

Therefore, particle charge is directly proportional to the square of particle radius, i.e. surface area.

According to Schneider [4], in the system with cylindrical configuration the value of charge on particles resulting from the disintegration of an electrified liquid jet is:

$$q = \frac{8\pi\varepsilon_0 V_0\ r^3}{3d_j^2 \ln\left(\dfrac{d_c}{d_j}\right)} \tag{5}$$

where V_0 the value of potential of the jet, d_c is the diameter of the surrounding cylinder and d_j is the diameter of the unperturbed jet. By taking into account the relationship

Manuscript received April, 18, 2009. The authors acknowledge with thanks the joint contributions of General Motors Canada Ltd and the Natural Sciences and Engineering Research Council of Canada for financial support in aid of this project.

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between particle radius and the jet diameter:

$$r^3 = \frac{3}{16} d_{\mathrm{j}}^2 \lambda \qquad (6)$$

where λ is the wavelength, it can be shown using Taylor expansion that the charge is practically linearly dependent on particle radius for the values of particle radius smaller than cylinder radius, i.e. n=1.

The value of electric field at the unperturbed jet surface is given by:

$$E_0 = \frac{2V_0}{d_{\mathrm{j}} \ln \frac{d_{\mathrm{c}}}{d_{\mathrm{j}}}} \qquad (7)$$

Using (6) and (7), the Equation (5) can be rewritten in the alternative form:

$$q = \frac{1}{\sqrt{3}} \pi \varepsilon_0 E_0 \lambda^{\frac{1}{2}} r^{\frac{3}{2}} \qquad (8)$$

Equating the right sides of (1) and (8) yields:

$$A = \frac{1}{\sqrt{3}} \pi \varepsilon_0 E_0 \lambda^{\frac{1}{2}} \qquad (9)$$

$$n = 1.5 \qquad (10)$$

Charge is proportional to the particle radius to the power of 1.5. Therefore, in the same system, the radius exponent may have one value if the voltage was kept constant and the other value if the electric field was kept constant.

For the idealized case of a spherical particle at the tip of a cone opposite a plane electrode, the value of acquired charge is [5]:

$$q = \frac{2\pi\varepsilon_0 V_0 (2 + 1/v)(r/l)^v r}{1 - (r/l)^{2v+1}} P_{v+1}(\cos\theta_0) \qquad (11)$$

where l is the distance between the sphere and the plane, V_0 is the applied voltage, θ_0 is the angle of cone originating from the centre of the sphere, $P_v(x)$ are the Legendre functions and v is the non integral index. Graphical representation of this equation shows practically linear charge to radius dependency.

Many workers have presented experimental results for the charge of conductive particles. Orme et al. [6] observed the formation of highly uniform charged molten metal particles from capillary stream breakup for applications requiring high-speed and high-precision deposition of molten metal particles, such as used in direct write technologies. Their experimental results showed that, in the first approximation, charge varies linearly with radius for the size ranges of practical interest. Krohn [7] examined the production of liquid metal particles for heavy particle propulsion and found the radius exponent to be about 1.3. Divey et al. [8] used ESPART to measure charge on particles produced by a spinning disk. Experimental data shows that charge to mass ratio was proportional to the particle radius to the power of -2 indicating the radius exponent was also one. In another study, a Phase Doppler Anemometry (PDA) technique in conjunction with a high-resolution computer-controlled traversing system was employed [9]. The experimental data showed that the particle charge also varies linearly with particle radius. This linear dependency conflicts with the established experimental and theoretical results for ionic charging of particles which predicts that charge is proportional to surface area or the square of the radius. Moore [10] presented a formula which predicts that conductive sphere in direct contact with a larger sphere attains charge that is proportional to the square of smaller sphere radius in agreement with the theoretical work of Felici [3]. Numerical simulations [11]-[13] also showed excellent agreement with Felici's formula given as (2) and predicted that the saturation charge of spherical particle conductively charged in a uniform electric field is proportional to the square of particle radius. Due to this conflicting data existing in the literature the current study was initiated in order to clarify the radius dependency for conductive particles charged in an electric field.

In this paper, the relation between the value of charge on a conductive liquid particle and the particle radius resulting from induction charging was investigated. The aim of the study is to establish the value of the unknown coefficient n, i.e. radius exponent in the charge to radius dependency, for the conductive particles atomized in the presence of an electric field. In the subsequent sections, the results of the numerical simulations for the induction charging of a conductive particle during a liquid atomization process are presented and a novel approach to the analytical treatment of an idealized approximation of the problem is introduced. Although a single conductive liquid particle formed in the ligament mode of atomization is studied, the general conclusions are also applicable to solid particles resting on a surface i.e. the case corresponding to a ligament length of zero. COMSOL, a finite element analysis software package, was employed for the numerical modeling. The numerical model takes into account the effects of the electrified bulk material undergoing atomization, the electrified ligament, the generated space charge cloud and the grounded electrode. In the analytical method, the method of images is used to accurately determine the influence of the presence of the electrified bulk material on the charging of the particle. In both approaches, the emphasis is given to the estimation of the radius exponent, n, in the particle charge relationship given by (1).

II. Theoretical Considerations

Although particles can acquire charge through tribo, corona, conduction and induction charging phenomena, induction charging is predominant for conductive particles. Induction charging refers to a process in which a voltage is connected to an electrode which is placed adjacent to the grounded conductive or semi-conductive material undergoing atomization. Electric charge flows from the ground to the material surface due to the induced electric field. In the separation process, the particles retain charge.

For solid particles, abrupt separation of the particle from a surface, normally in the form of packed powder, is the only possible induction charging mechanism. For liquid atomization, the bulk material goes through a complex disintegration phase prior to particle separation. The liquid flowing out from the atomizer either forms a meniscus, which becomes elongated into jets and further disintegrated into droplets, or a liquid sheet which breaks into ligaments (due to the unstable waves formed at the liquid-gas interface) and further disintegrates into particles. There are two groups of forces which cause deformation and disruption of the liquid: bulk forces in the liquid, and normal and tangential stresses at the liquid surface. In practically all the commercial atomizers the governing atomization mode is ligament disintegration [14]. Ligament breakup is depicted in the Figure 1 [15].

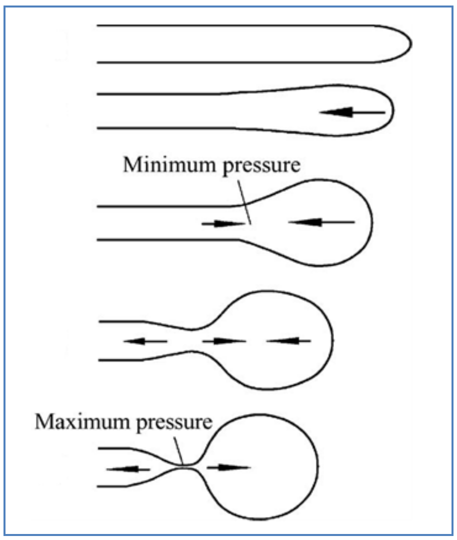

Fig. 1. Ligament breakup [15].

In the initial stage, surface tension due to the curvature of the rounded tip of the ligament creates a pressure gradient. This causes the liquid to move backwards which results in creation of a spherical region at the ligament tip. A minimum pressure point is created and the flow is directed towards the minimum pressure point which generates a typical neck profile. Due to the curvature of the liquid surface in the neck formation region, a maximum pressure point is generated. The liquid flows away from the neck region resulting in an increase in volume of the sphere region and further decrease of the neck region. This unstable situation leads to final separation of the spherical droplet.

III. Numerical Estimation of Q/M Ratio Distribution

A. *Description of the Numerical Procedure*

The problem of particle charging can be efficiently simulated numerically using COMSOL, a finite element analysis software package. The electric potential distribution is governed by the Poisson equation [16]:

$$-\nabla^2 V = \frac{\rho}{\varepsilon} \tag{12}$$

where V is the electric potential, ρ is the space charge density and ε is the electric permittivity.

Assuming that the material undergoing atomization is highly conductive, we can consider that all the charge transfer between the bulk material and the particles happens instantaneously. This enables us to use a static approach when modeling the charging process.

The computational domain for the specified problem encompasses the simplified atomizer (bulk material), space charge cloud, the target, ligament and the particle at the moment of the ligament-particle separation. Figure 2 represents the particle and the ligament of width r_{lig} and length of l. The computational model generated in this manner represents the liquid material atomization for greater values of ligament length, and solid material charging for the case of very small values of ligament length.

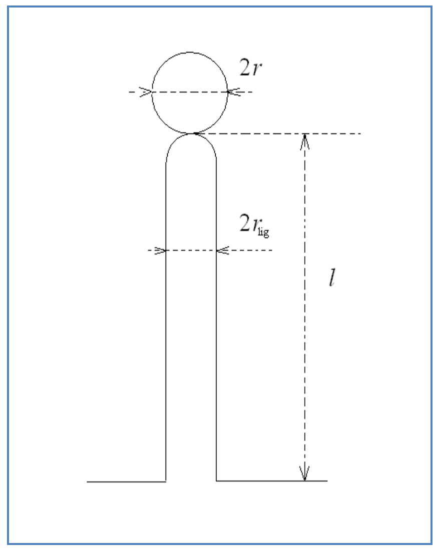

Fig. 2. Particle and the ligament in the moment of particle-ligament separation.

In this model the atomizer, ligament and the particle are considered directly connected to the power voltage supply, whereas the target is grounded. The base values of the input parameters for the numerical model are obtained from experimental work [14] and are summarized in the following table.

TABLE I
Parameters of the Computational Model

Parameter	Base Value
Atomizer radius	100mm
Ligament length	2mm
Ratio of particle and ligament radii	1.5
Particle radius	15µm
Volume space charge	-25µC/m^3
Applied voltage	-90kV
Target distance	0.25m

B. *Results of the Numerical Modeling*

In order to find out which parameters affect the radius exponent for the particle charge to radius dependency, a series of numerical simulations were carried out. Ligament length was varied in range between 0 and 4mm and the value of charge calculated. For a particle of radius r_1 with the charge level of q_1, and a particle of radius r_2 with the charge level of q_2, (1) yields respectively:

$$q_1 = Ar_1^n \tag{13}$$

$$q_2 = Ar_2^n \tag{14}$$

Dividing (13) by (14) gives:

$$\frac{q_1}{q_2} = \left(\frac{r_1}{r_2}\right)^n \tag{15}$$

From (15), parameter n can be calculated as:

$$n = \frac{\log \frac{q_1}{q_2}}{\log \frac{r_1}{r_2}} \tag{16}$$

Figure 3 shows the plot of particle charge versus the normalized ligament length for the values of the parameters listed in Table 1. The particle charge increases monotonically and almost linearly as the ligament length increases. Although not clear from the graph, the *y* intercept corresponds to a charge of -0.015pC which agrees well with the value predicted by (2).

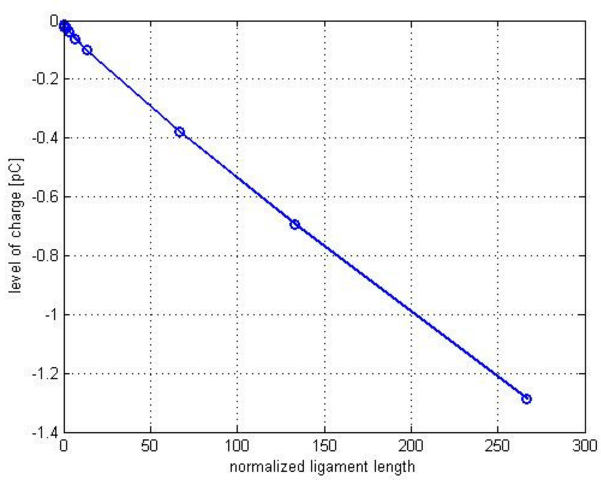

Fig. 3. Particle charge versus normalized ligament length for a particle with r=15μm and the values of parameters as in Table 1.

Figure 4 shows the plot of the radius exponent, n, as derived from (16) versus the normalized ligament length for the values of the parameters listed in Table 1. The radius exponent is practically equal to two when the particle is in direct contact with the atomizer.

The radius exponent decreases rapidly as the ligament length is increased. For the operational range of ligament length (normalized value of approximately 130) the value of radius exponent is practically equal to 1.1, but further decreases for increasing ligament length approaching an asymptotic value of 1.08.

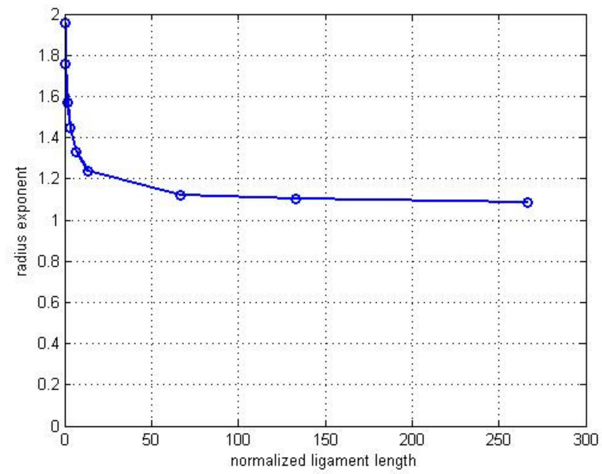

Fig. 4. Radius exponent versus normalized ligament length for the values of parameters as in Table 1.

Various factors can potentially affect the level of particle charge: the space charge formed between the atomizer and the target, the presence of previously formed droplets, and the presence of adjacent ligaments, such as found in commercial atomization systems. In addition, in order to see effect of the presence of the ligament the hypothetical case in which the ligament is absent has also been modeled. The overlapped graphs that illustrate the effects of these parameters on the level of particle charge and radius exponent, n, are shown in Figure 5 and Figure 6, respectively. For examining the effect of space charge, the volume space charge is doubled to the value of -50μC/m^3. The increase in the value of volume space charge results in a decrease of the value of particle charge. This is a direct consequence of the space charge reducing the electric field strength at the point of atomization. Analogously, a decrease in the value of volume space charge results in an increase of the value of particle charge.

The influence of previously formed particles on the charging process of an atomizing particle was examined by placing a uniformly charged particle 30μm above the point of atomization. The presence of this previously formed particle slightly decreases the value of the original particle charge. Again, this is a direct consequence of the reduction of the charging field strength. Figure 6 also suggests that neither the presence of space charge nor the previously formed particle has any practical effect on the value of the radius exponent.

A theoretical case when the ligament is removed from the model was also studied. The absence of the ligament increases the value of the particle charge because the shielding effect becomes less pronounced. The shape of the plot is, however, essentially unaltered compared to the base case. Figure 6 suggests that the absence of the ligament substantially reduces the value of the radius exponent *n* and it reaches a limiting value of *n*=1. This is because the ligament introduces a shielding effect which results in a reduction of the effective distance between the bulk material and the particle and causes the shifting of the curve.

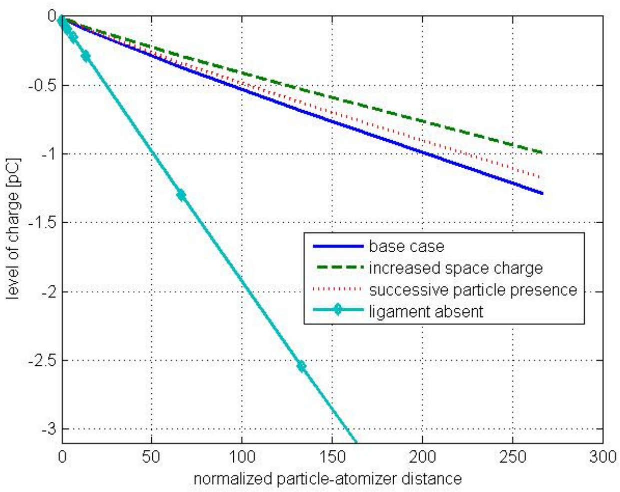

Fig. 5. Particle charge versus normalized distance between the atomizer and the particle for different configurations of the system.

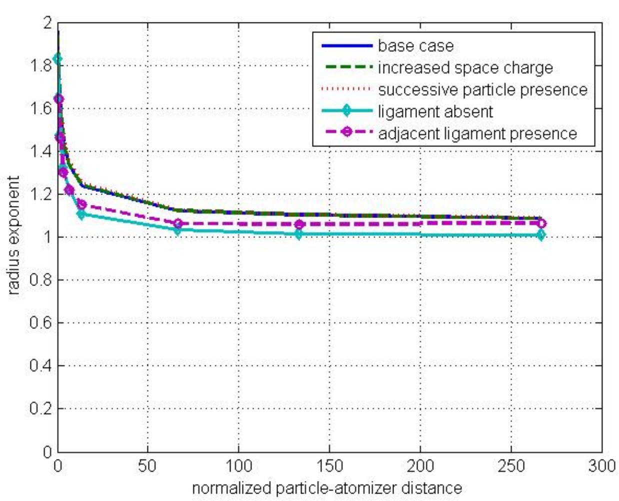

Fig. 6. Radius exponent versus normalized distance between the atomizer and the particle for the base and derived cases.

To examine the effect of the ligament spacing, a simplified three dimensional model was generated. The model encompasses a series of electrified ligaments with 50μm spacing placed in linear array. Because of the meshing problem associated with the high aspect ratio of this geometry, the size of the domain needed to be significantly reduced. However, it was found that the presence of adjacent ligaments also reduced the value of the particle charge due to the shielding effect. This effect is not very pronounced suggesting that at the assumed spacing the mutual coupling between the ligaments is not strong. Considering the modeling and numerical errors, it can be noted (based on the Figure 6) that the value of the radius exponent, *n*, for the 3D model follows well the corresponding value for the base case, but the graph is positioned somewhat lower.

C. Approximation of the Numerical Results

The following formula has been used for fitting of the numerical results:

$$Q = \left(K_1 r^{0.1}\right)\left(\frac{l}{r}\right) r^2 + K_2 r^2 \tag{17}$$

where K_1 and K_2 are experimentally found constants.

For the case without space charge where the ligament length takes values from 0 to 4mm, the numerical and corresponding fitted results for the value of charge on the particles of radii 15μm, 30μm, and 45μm are shown in Figure 7. As expected an increase in particle radius and/or an increase in ligament length results in larger particle charge.

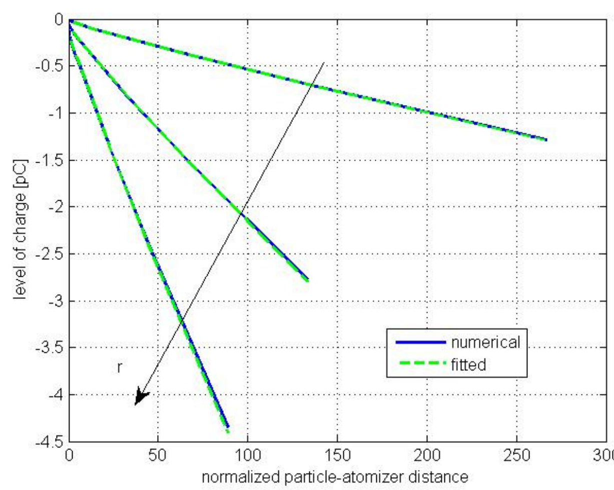

Fig. 7. Comparison of numerical results and fitted data for the value of charge as a function of ligament length (*r*=15, 30, 45μm).

The approximation and the correct, analytical solution are in very good agreement. For this case, the value of constant K_1 is -39.3μC/m, whereas the value of constant K_2 is -64.78μC/m^2. The value of constant K_2 is practically identical to $6.56\pi\varepsilon E_0$, and can be predicted using Felici's formula [3] for the conductive sphere touching the electrified plane. Here, E_0 is the external electric field which, in this case, takes value of -346kV/m.

For all of the above mentioned cases, the highest level of the electric field occurs at the point of the particle surface which is closest to the grounded target. The well known Peek's formula [17] was used to calculate the threshold level of the electric field for the onset of corona discharge. The threshold level was then compared to the numerical value of the highest electric field. It was found that for all studied cases the corona discharge does not occur in the operational range.

IV. Analytical Estimation of Charge to Radius Dependency

A. Description of the Analytic Method

An exact analytical solution for the previously given geometry is not possible. A further simplification is required. For this purpose, a spherical conductive particle of radius a situated at the distance l above the atomizer modeled as a conductive sphere of radius b ($b>>a$) is considered as shown in Figure 8.

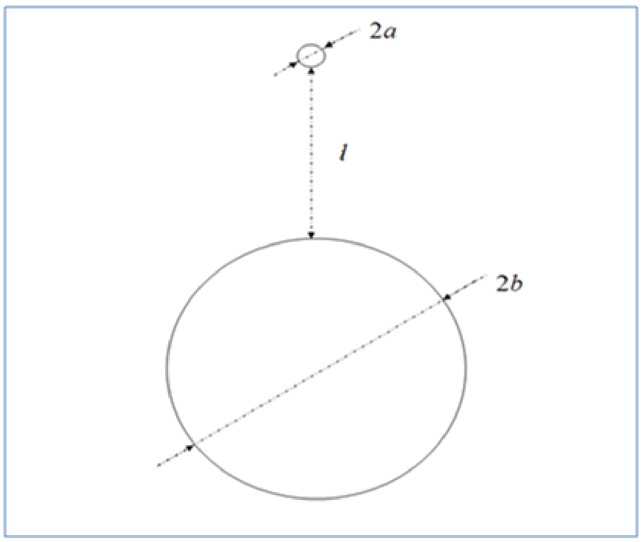

Fig. 8. Two sphere geometry.

Both the particle and the atomizer are kept at the same potential V and the method of images is employed. The particle acquires charge Q which can be expressed using Maxwell coefficients as:

$$Q=C_{11}V+C_{12}V, \quad (18)$$

where C_{11} is given by the formula [18]:

$$C_{11}=4\pi\varepsilon_0 ab\sinh\alpha\cdot \sum_{m=1}^{\infty}\left(b\sinh m\alpha+\alpha\sinh(m-1)\alpha\right)^{-1}, \quad (19)$$

and C_{12} is given by the formula:

$$C_{12}=-4\pi\varepsilon_0\frac{ab}{c}\sinh\alpha\sum_{m=1}^{\infty}\left(\sinh m\alpha\right)^{-1}. \quad (20)$$

ε_0 is the electric permittivity of the vacuum and α is evaluated by:

$$\alpha=\operatorname{arccosh}\left(\frac{c^2-a^2-b^2}{2ab}\right) \quad (21)$$

where

$$c=l+a+b. \quad (22)$$

B. Results of the Analytical Approach

The value of charge is calculated based on (18)-(22) for the voltage of -90kV, the atomizer radius of 100µm, the particle radius of 15µm and for different values of particle-atomizer distance. Equation (16) gives the radius exponent dependency of the normalized particle-atomizer distance as shown in Figure 9.

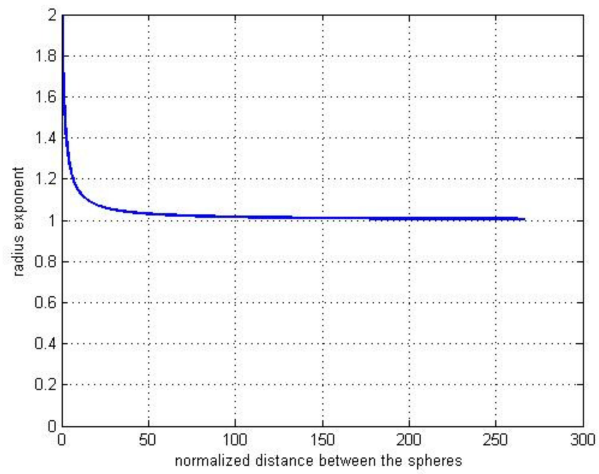

Fig. 9. Radius exponent versus normalized sphere distance for the analytical model.

Only at very small distances between the atomizer and the particle, the charge of the particle is approximately proportional to the square of the particle radius. For greater distances, the charge of the particle is approximately proportional to the particle radius. The curves depicted on Figure 4 and Figure 9 are in a good general agreement with the important difference that in Figure 9 the exponent reaches exactly one whereas in Figure 4 it never falls below 1.08. This suggests that the distance between the electrified bulk material and the particle is the main parameter that has any practical influence on the radius exponent in the charge to radius dependency. However, the difference between the numerical and the analytical model is a result of the absence of the ligament in the analytical model which changes the electric field configuration in the vicinity of the particle. This is confirmed by the concurrence of Figure 9 with the case of the ligament absent in Figure 6.

C. Approximation of the analytical formula

The hyperbolic functions used in the derivation of the analytical formula are not suitable for practical calculations. An approximation of the analytical formula is, therefore, desirable.

The following formula (that does not use the hyperbolic functions) has been used for data fitting:

$$Q = K_1\left(\frac{l}{r}\right)r^2 + K_2 r^2 \quad (23)$$

Figure 10 compares the analytical and the fitted curves based on (18) and (23), respectively, for the value of charge in function of distance between spheres for particle radii of 15µm, 35µm and 45µm.

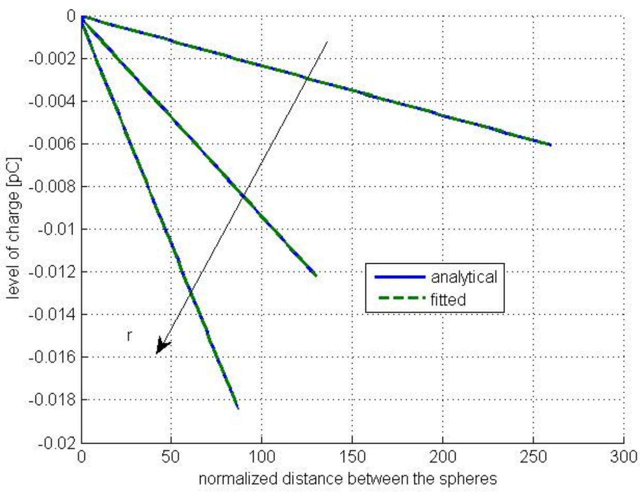

Fig. 10. Comparison of analytical and fitted curves for the value of charge as function of distance between spheres.

The approximation and the correct, analytical solution are in very good agreement. In this particular studied case, constants K_1 and K_2 take values 0.103nC/m and 0.16µC/m^2 respectively. Also, it can be noted that the values of charge for the analytical solutions are several order of magnitude smaller than the values of charge for the numerical solution. This difference is caused by the absence of the grounded plane in the analytical solution.

V. Discussion

Several important observations can be made based on the presented results. The particle charge to radius dependency is quadratic when the particle is in direct contact with the electrified material and becomes almost linear when the distance between the particle and the material is increased. This can explain the fact that in previously published theoretical and experimental findings, solid particle charges in majority of cases were found to be proportional to the particle surface area, whereas for the liquid particles in the majority of cases charges were practically linearly dependent on particle radius. Interestingly, some results [7] show a dependency of approximately 1.3, a result predicted here if the ligament length is of the order 10 times the radius. The dependency is very robust and practically independent on variations of typical parameters such as voltage, space charge and the presence of additional electrified or charged objects. This dependency is confirmed by the analytical solution based on the method of images. The difference between the numerical simulation and the analytical solution at greater distances is a result of the absence of the ligament in the analytical solution. Another important observation is that the liquid particle charge increases linearly with the increasing of the ligament length. Theoretically, the charge would increase indefinitely if the external electric field is uniform. However, in reality, particle charge will eventually reach a saturation level due to breakdown. It is interesting to note that although the values of the charge in the study are relatively high, for the values assumed the corona onset level was not reached and discharge did not occur. Finally, although the actual geometrical problem is too complex to be solved using exact analytical methods, the final results can be fitted very well to a simple polynomial formula which is very convenient for any further analysis.

VI. Conclusions

In this paper, the relation between the value of charge for a conductive particle and the particle radius in the process of induction charging was investigated. The aim of the study was to establish the value of the radius exponent in the charge to radius dependency for the conductive particles atomized in an electric field. COMSOL, a finite element analysis software package, was employed for the numerical modeling. It was found that the single most predominant parameter affecting the particle charge to radius dependency is the ligament length. The radius exponent is equal to two when the particle is in the direct contact with the bulk of material undergoing atomization. This agrees with the results of the previously published papers for the conductive solid particles for which the charge is proportional to the surface area. The radius exponent decreases rapidly as the ligament length is increased. For the distances greater than the particle radius, the radius exponent is approximately equal to one but is never exactly one because of the presence of the ligament. This also agrees with the results of the previously published papers for the liquid particles in the ligament mode atomization where the charge is proportional to particle radius. This study therefore, resolves some of the conflicting data in the previously published literature. Also, it was found that particle net charge increases in direct proportion to the ligament length which could have important implications in electrostatic coating applications. Agreement between numerical and analytical results based on the method of images was found to be very good. This further justifies use of the numerical method which is preferred option compared to analytical and experimental methods due to the ease of modeling of physical problems.

Three significant practical conclusions resulting from this work are:

1) For electrostatic coating applications using inductively charged solid particles the charge to mass ratio is proportional to 1/r,
2) For electrostatic coating applications using ligament atomization of liquids the charge to mass ratio is approximately proportional to $1/r^2$
3) The maximum charge to mass ratio in liquid spraying will be achieved by maximizing the ligament length.

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