Electrostatic Sticking of Sheets Fed from a Stack

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***Abstract*—In sheet feeding applications such as printing, sheets are typically fed from the top of an input stack. In one common failure mode, the first few sheets feed reliably from a fresh stack. However, as feeding continues and the input stack grows shorter, sheets fed from the middle of the original stack begin to stick to the top of the now shorter stack. A theoretical analysis identifies a mechanism to explain this feeding failure. The electric fields in the air gaps between the sheets are predicted to increase with depth into the stack. When these air gap fields exceed the Paschen limit, air ionization occurs that results in charge separation across the air gaps. This charge separation causes strong electrostatic attraction between the sheets, which results in sticking. To prevent this feeding failure, the electrostatic surface charge density on the sheets must be kept below a critical value when the sheets are stacked during converting and packaging phase of their manufacture. The theoretical maximum allowable surface charge density $\rho_{S,MAX}$ is on the order of $3\,\mu C/m^2$ for a stack of 100 sheets. To allow a margin of robustness, the specification limit for maximum charge on a sheet should be 10X below this theoretical value; $\rho_{S,LIMIT} < 0.3\,\mu C/m^2$. For stacks with more sheets, charge density $\rho_{S,MAX}$ is lower. Charge density $\rho_{S,MAX}$ varies with the air gap between the sheets, which is determined by the surface roughness of the sheet material and by the flatness of the sheets. Smooth, flat sheets can tolerate a somewhat higher surface charge density.**

***Index Terms*—electric breakdown, electric force, charge density, Paschen limit**

I. Introduction

Feeding sheets from an input stack is a critical operation in printers, medical imagers, document scanners, copiers, currency handling in ATM machines, and many other applications. In each case, a sheet must be feed from an input stack, transported through a value adding process such as printing, scanning, or counting, and collected in an output stack. Sheet transport and handling involves many engineering challenges because individual sheets must be moved at high speed by rollers and belts, registration must be maintained by guides and sensors, and the sheet must not be damaged, curled or skewed during transport. And, the process must be robust to variability in the input sheets, whose physical properties (thickness, flatness, surface roughness, etc) and electrical properties (conductivity, surface charge density) vary.

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One of the operations highly prone to failure is feeding sheets from the input stack. Many of us who have tried to make copies of a document just before an important meeting have experienced the frustration of feeding failures and the resulting sheet jams. Often, the process begins normally and runs well. However, as the process continues, when the mechanism tries to feed sheets from the middle of the original input stack, sheets stick together resulting in double feeds or no sheet being feed at all. The countermeasure often used is to fan the sheets remaining in the input stack, which sometimes works. A better solution is to simply replace the partially expired input stack with a new stack.

The root cause of this problem is static charge on the sheets when they are stacked during the manufacturing process. In this paper, a failure mechanism is described that explains why the first few sheets feed reliably and why sheet sticking occurs only after some sheets have been fed. Fanning the partially expired stack does affect the charge density on the sheets, though it is not clear that the sheet sticking problem can be solved by fanning. The best counter measure is to replace the input stack by a fresh stack.

The solution to this failure mode is to reduce the static charge on the sheets when they are stacked during the manufacturing process. The surface charge density on the sheets must be lower than a maximum allowable level to prevent sheet sticking during feeding operations. The critical surface charge density in on the order of $3\mu C/m^2$ for a stack of 100 sheets, and the charge density limit decreases as the number of sheets increases. A somewhat higher charge density can be tolerated on smooth, flat sheets with low surface roughness.

II. Analysis

A. Electrostatic Attraction Between Sheets

Coulombic attraction between charged sheets is one root cause of sticking. In Fig. 1, the positive charge on the upper sheet is attracted to the equal and opposite charge on the lower sheet that is separated by an air gap of distance δ. Gauss' Law (1) can be used to determine the electric field E_{GAP} within the air gap between the sheets (2).

$$\oint\!\!\oint_S \varepsilon_0 \vec{E} \cdot d\vec{s} = \iiint_V \rho \, dv \quad (1)$$

$$E_{GAP} = \frac{\rho_S}{\varepsilon_0} \quad (2)$$

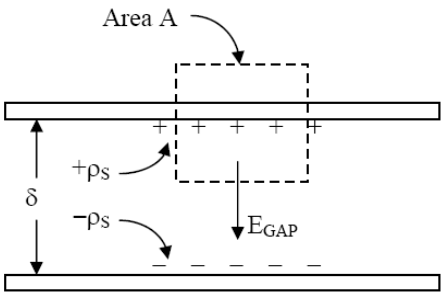

Fig. 1. A patch of equal and opposite charge across an air gap between adjacent sheets causes strong electrostatic attraction (sticking).

The increase in electric energy (3) as the air gap spacing increases is due to the electric force as in (4).

$$W^e = \iiint_V \frac{1}{2}\varepsilon_0 E^2 dv = \frac{1}{2}\varepsilon_0 \left(\frac{\rho_S}{\varepsilon_0}\right)^2 A\,\delta \tag{3}$$

$$f^e = -\frac{\partial W^e}{\partial \delta} = -\frac{\rho_S^2}{2\varepsilon_0} A \tag{4}$$

The electric force per unit area or electric pressure (5) is plotted in Fig. 2.

$$p^e = \frac{f^e}{A} = -\frac{\rho_S^2}{2\varepsilon_0} \tag{5}$$

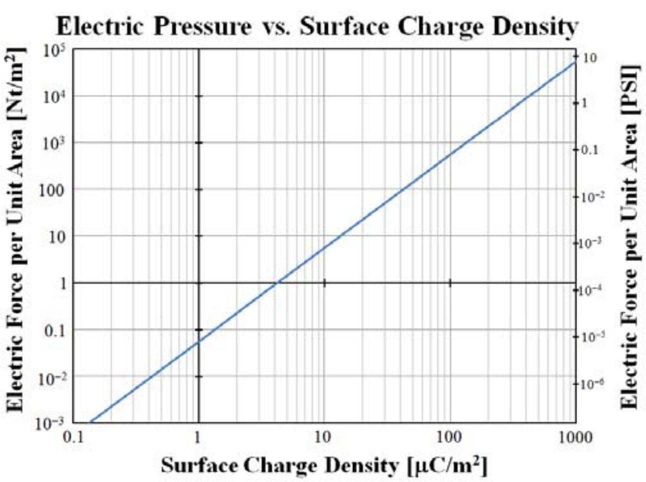

Fig. 2. The electric force per unit area varies with the square of the surface charge density.

Two results of this analysis are:

1. The electrostatic force per unit area is proportional to the square of the charge density.
2. Feeding failures occur when the pressure between sheets exceeds about $10^{+4}\,\mathrm{Nt/m^2}$. Charge patches with a surface charge densities exceeding about $400\,\mu\mathrm{C/m^2}$ are sufficient for electrostatic attraction strong enough to cause sheets to stick.

B. Electric Field In The Air Gaps Between Sheets

The stack of sheets in Fig. 3 is resting on a grounded plate. Each sheet in a stack has an electric surface charge density ρ_S. Gauss' Law is used to find the electric field $E_{GAP,n}$ across the air gap below the n^{th} sheet to be (6).

$$E_{GAP,n} = n\left(\frac{\rho_S}{\varepsilon_0}\right) \tag{6}$$

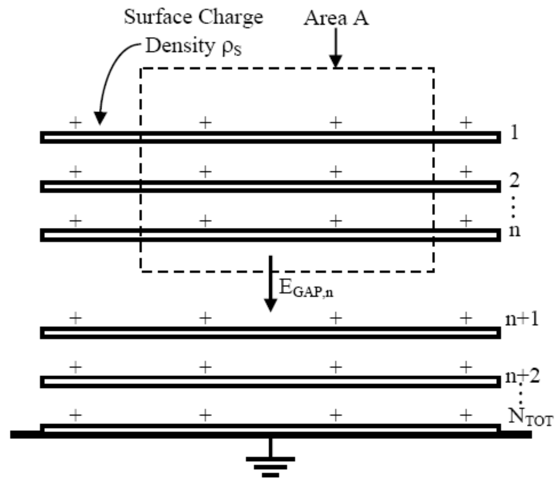

Fig. 3. The electric field in the air gap between the sheets increases when a charged sheet is added to the top of the stack.

The electric field in the gap between the sheets is proportional to the number of charged sheets above the air gap. The field across each air gap in the stack increases when a charged sheet is added to the top of the stack. Assuming that the electric field is approximately uniform or that the spatial variation of the charge density is small over the distance of an air gap, the voltage across an air gap is estimated in (7).

$$V_{GAP,n} \cong n\,\delta\left(\frac{\rho_S}{\varepsilon_0}\right) \tag{7}$$

For an air gap of 5 μm in a stack of 10 sheets, the surface charge density resulting in the Paschen limiting voltage of about 360 V is (8).

$$\rho_S \cong \frac{\varepsilon_0 V_{GAP,n}}{n\,\delta} = \frac{(8.854\,\mathrm{pF/m})(360\,\mathrm{V})}{(10)(5\,\mu\mathrm{m})} = 63.7\,\mu\mathrm{C/m^2} \tag{8}$$

The voltage across each air gap and the charge density transferred by electrical discharge are plotted in Fig. 4 for the stack analyzed in (8).

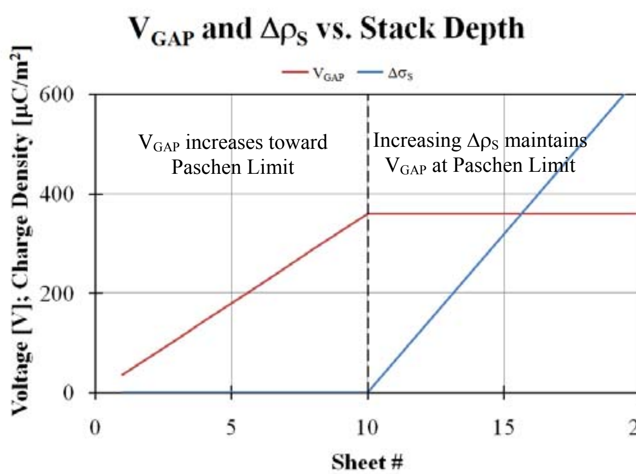

Fig. 4. The voltage across the air gap below each sheet increases because each sheet is charged. The voltage is limited by air breakdown to the Paschen limit of about 360V by charge $\Delta\rho_S$ transferred across the air gap.

C. Breakdown In Air Gaps Between Sheets

Once the electric field $E_{GAP,n}$ reaches the Paschen limit, adding another charged sheet to the top of the stack as in

Fig. 5 initially increases the electric field that causes air discharge that transfers charge across the gap onto the surfaces of the sheets. The transferred charge reduces the field and maintains the field at E_{LIMIT}.

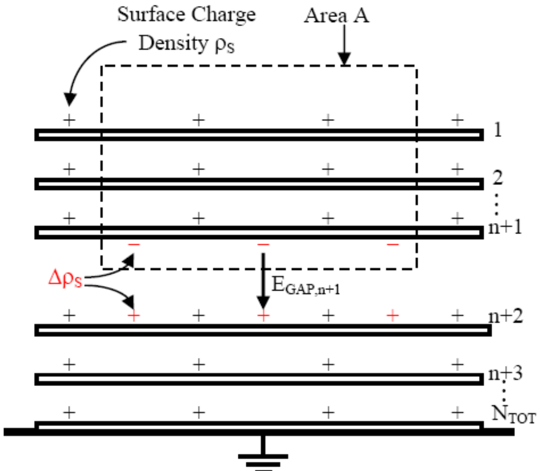

Fig. 5. Breakdown in the air gap between the sheets limits the electric field to the Paschen limit.

Gauss' Law is used in (9) to find the electric field $E_{GAP,n+1}$ in the air including the charge density $\Delta\rho_S$ deposited on the sheet surfaces by the electrical breakdown in the air gap.

$$E_{GAP,n+1} = (n+1)\left(\frac{\rho_S}{\varepsilon_0}\right) - \frac{\Delta\rho_S}{\varepsilon_0} \tag{9}$$

Knowing that the electric field in the gap is maintained at the Paschen limit, the amount of the charge density $\Delta\rho_S$ deposited on the surfaces of the sheets by the electrical breakdown in the air gap is (10).

$$\Delta\rho_S = \rho_S \tag{10}$$

When a sheet with surface charge density ρ_S is added to the top of the stack, an equal amount of charge is deposited on the surfaces of sheets in the stack that have previously experienced a discharge. Additional charge accumulates on the sheets with each sheet added to the top of the stack. This accumulated charge can grow to become very high.

D. Feed 1 Sheet At A Time From Top Of Stack

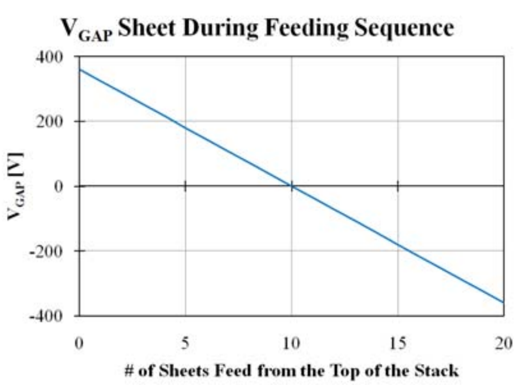

Fig. 6. Voltage V_{GAP} is initially at the Paschen limit (~360V) and decreases steadily as sheets are fed until it again reaches the Paschen limit with the opposite polarity.

The feeding sequence where the top sheet is fed from stack is now analyzed. When these charged sheets were stacked, electrical discharges occurred within the air gaps. The voltage V_{GAP} across the air gap below the $(2n)^{th}$ sheet (Fig. 5) is computed and V_{GAP} for this gap is plotted in Fig. 6 after each sheet is fed. Prior to feeding the first sheet, the voltage is (11).

$$V_{GAP,2n} = (2n)\delta\left(\frac{\rho_S}{\varepsilon_0}\right) - n\,\delta\frac{\Delta\rho_S}{\varepsilon_0} \tag{11}$$

Voltage VGAP is at the Paschen limit because air discharge has occurred in the air gap. The consequence of the discharges is that surface charge density $n\delta\rho_S$ has accumulated on the surface of the sheet. This surface charge density remains constant during the feeding sequence.

After the first n sheets are feed from the top of the stack, the voltage across the same air gap below the now n^{th} sheet is approximately zero as in (12).

$$V_{GAP,n} = (n)\delta\left(\frac{\rho_S}{\varepsilon_0}\right) - n\,\delta\frac{\Delta\rho_S}{\varepsilon_0} \approx 0 \tag{12}$$

The electric field from the charge density deposited from the n previous air discharges opposes the electric field from the n sheets in the stack and the resultant field is nearly zero.

During this initial phase of the feeding sequence, the charge density on the lower surface of the top sheet has been too low to cause sheet sticking. As the feeding sequence continues, the next (n+1) sheets are fed leaving just the original $(2n)^{th}$ sheet to be fed. The voltage across the air gap below the now top sheet is nearly at the Paschen limit as in (13).

$$V_{GAP,n} = \delta\left(\frac{\rho_S}{\varepsilon_0}\right) - n\,\delta\frac{\Delta\rho_S}{\varepsilon_0} \approx -(n-1)\delta\left(\frac{\rho_S}{\varepsilon_0}\right) \tag{13}$$

During this phase of the feeding sequence, the voltage across the air gap increases towards the Paschen limit because the air breakdowns that occurred when the stack was formed accumulated higher levels of charge density within the stack.

E. Fanning The Stack

Prior to the feeding sequence, the charged stack is in the state shown in Fig. 5 where the voltage across the air gap below the $(2n)^{th}$ sheet is (14).

$$V_{GAP,2n} = (2n)\delta\left(\frac{\rho_S}{\varepsilon_0}\right) - n\,\delta\frac{\Delta\rho_S}{\varepsilon_0} \tag{14}$$

To determine the effect of fanning the stack, let the air gap δ become very large. Initially, the voltage increases linearly with increasing δ. However, $V_{GAP,2n}$ is already at the Paschen limit because air breakdowns occurred in this gap when the stack was formed. By increasing the air gap, the voltage increases causing further air breakdown. Audible crackling is often apparent when fanning a stack of charged sheets. This air breakdown further increases the charge density on the surfaces of the sheets in an uncontrolled and chaotic manner. While the resulting charge distribution is difficult to predict, the air breakdown deposits additional charge on the sheet surfaces. Reliable feeding is not guaranteed.

III. Discussion

Analysis of charged sheets in a stack shows that the surface charge density is limited by the Paschen Limit and that charge densities at this limit exist within the stack. The modified

Paschen voltage is plotted in Fig. 7. The resulting electric field E and surface charge density ρ_S are also plotted. The Paschen voltage is modified because at very small spacing, the electric field becomes so high that charge is transferred across the gap by non-linear conduction mechanisms such as field emission. The Paschen voltage is modified to include this non-linear conduction by limiting the electric field to $75\,\mathrm{V/\mu m}$ when the gap is less than 5µm. The resulting expression for the Paschen voltage (15) is a piece-wise continuous function combining several expressions valid over different electrode spacing [1].

Given the voltage and spacing between two surfaces, the charge density is (16).

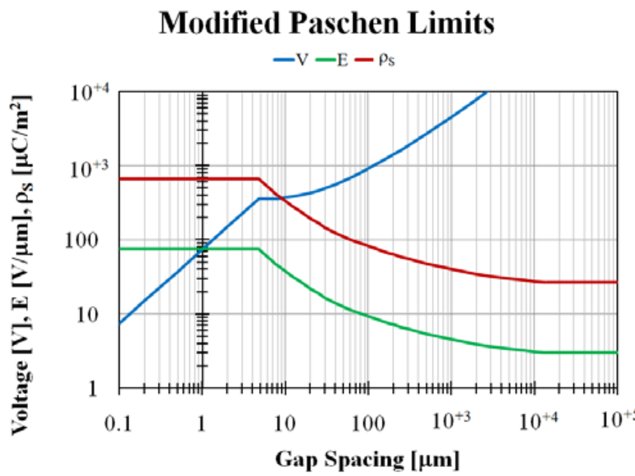

Fig. 7. In the modified Paschen curve, the voltage increases with gap except for a plateau between about 5 – 10µm. The electric field is limited to 75V/µm for gaps less than 5µm.

$$V_{PASCHEN}(d) = \begin{cases} \left(75\dfrac{V}{\mu m}\right)\cdot d[\mu m] & ;\quad 0 \le d \le 4.8\,\mu m \\ 360[V] & ;\quad 4.8 \le d \le \dfrac{48}{6.2}\,\mu m \\ 360[V] + 6.2\left[\dfrac{V}{\mu m}\right]\left(d[\mu m] - \dfrac{48}{6.2}[\mu m]\right) & ;\quad \dfrac{48}{6.2} \le d \le 100\,\mu m \\ 2.44\left[\dfrac{V}{\mu m}\right]\cdot d[\mu m]\cdot 65.3\left[\dfrac{V}{\sqrt{\mu m}}\right] + \dfrac{3500[\mu m\, V]}{d[\mu m]} & ;\quad 100 \le d \le 13598\,\mu m \\ 3\left[\dfrac{V}{\mu m}\right]\cdot d[\mu m] & ;\quad 13598\,\mu m \le d \end{cases} \tag{15}$$

$$\rho_S = \frac{\varepsilon_0 V_{GAP}}{\delta} \approx \frac{\left(8.854\dfrac{pF}{m}\right)(360V)}{(5\,\mu m)} = 637\,\frac{\mu C}{m^2} \tag{16}$$

For a voltage difference at the Paschen limit across a gap of 5µm, Fig. 2 shows that the electrostatic attraction is sufficient to cause sheets to stick.

To prevent electrostatic sheet sticking, the charge density must be kept sufficiently low to prevent electric breakdown when the sheets are stacked during the converting process. The voltage across each air gap in the stack must be kept below the Paschen limit. The maximum voltage is in the gap between the bottom sheet and the grounded plate. Using (7), the voltage limit is (17).

$$V_{GAP,N} \cong N_{STACK}\,\delta\left(\frac{\rho_S}{\varepsilon_0}\right) < V_{PASCHEN} \tag{17}$$

The limit on charge density (17) is plotted in Fig. 8. Several features of Fig. 8 are important.

1. The maximum permitted charge density decreases with more sheets in a stack.
2. The maximum permitted charge density is somewhat higher for smoother sheets.
3. For a typical stack of 100 sheets with a 10 µm surface roughness, the theoretical maximum charge density is about $3\mu C/m^2$. To provide a margin of safety for practical applications, the specification on charge density should be 10X lower than the theoretical maximum.

IV. Conclusion

In one common failure mode of sheets fed from an input stack, the feeding sequence begins properly and failure occurs after several sheets are successfully fed. A theoretical analysis shows that electrical breakdown in the air gaps between the sheets would cause feeding failures from the middle of a stack even though the first few sheets would experience little electrostatic sticking. Fanning the stack would certainly affect the charge density on the sheets, though it is unclear that reliable feeding would result. Rather, the solution is to limit the charge density on the sheets below a maximum permitted level $\rho_{S,MAX}$ when the stack is formed in the converting and packaging phase of manufacturing. For a typical stack of 100 sheets with a 10 µm surface roughness, the theoretical maximum charge density is about $3\mu C/m^2$. To provide a margin of safety for practical applications, the specification on charge density should be 10X lower than the theoretical maximum; $\rho_{S,LIMIT} < 0.3\mu C/m^2$. The maximum permitted charge density $\rho_{S,MAX}$ decreases with more sheets in a stack.

And, $\rho_{S,MAX}$ is somewhat higher for smoother sheets.

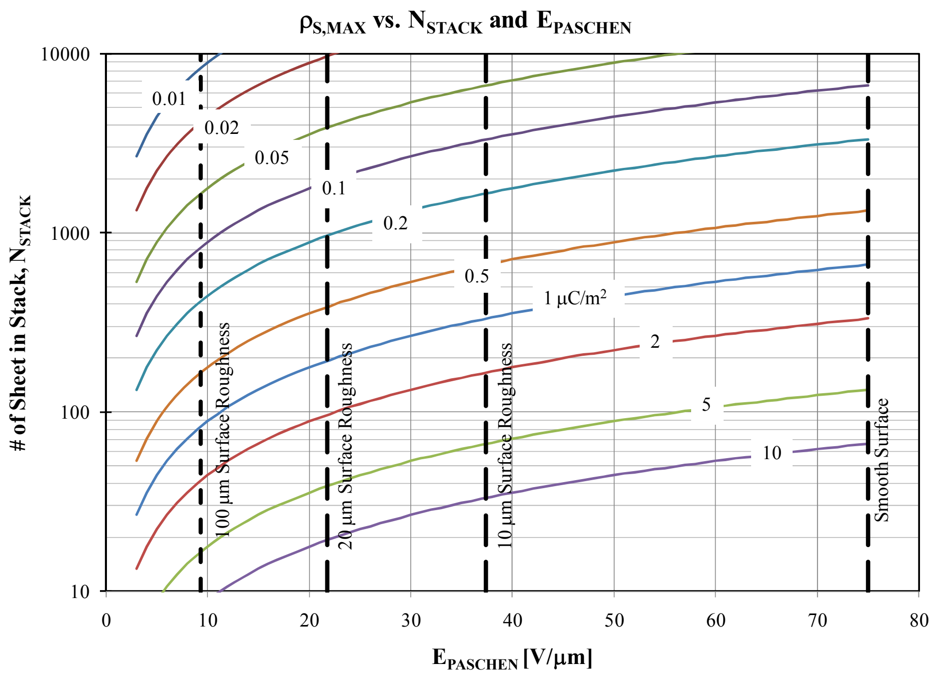

Fig. 8. For a stack of N_{STACK} sheets, the maximum charge density $\rho_{S,MAX}$ increases with increasing $E_{PASCHEN}$.

REFERENCES

[1] R. M. Schaffert, "14.3 The Modified Paschen Curve," electrophotography, London, Focal Press, 1975, pp. 516.

Kelly Robinson (M'78–SM'89) earned a BS in Engineering Science in 1976 from Colorado State University in Fort Collins, and an MS in Electrical Engineering in 1978 from the University of Illinois in Champaign Urbana. He earned the PhD in electrical engineering in 1982 from Colorado State University.

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Dr. Robinson served as the IEEE-IAS Electrostatics Processes Committee Chairman from 1987-88 and President of the Electrostatics Society of America from 2005-08.